

Automorphisms of twisted Chevalley groups over rings

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Abstract. Let R be a commutative ring with identity, equipped with an automorphism θ of order dividing 2 or 3. We assume that $2 \in R^\times$ and, in the triality case, also that $3 \in R^\times$. I will present a local–global classification of automorphisms of elementary and full twisted Chevalley groups of types ${}^2A_\ell$, $\ell \geq 3$, ${}^2D_\ell$, $\ell \geq 4$, 2E_6 , and 3D_4 , for every invariant isogeny type.

The proof combines the normal structure of twisted Chevalley groups with local rigidity and affine gluing. An automorphism first acts on the maximal congruence quotients. After a global normalization of the standard relative-root frame, its action on the relative root subgroups is recovered from an automorphism of the coefficient ring. Consequently, outside a small list of exceptional cases, all automorphisms are compositions of the familiar inner, diagonal, ring, graph, and central factors.

Over arbitrary rings two additional phenomena occur. In the low-rank unitary types 2A_3 and 2A_4 , there is an additive family of exceptional automorphisms supported on the fixed residual characteristic-three locus. In absolute type D_4 , local graph data must be assembled into compatible global graph–ring automorphisms. In triality, the coefficient factor is governed by preservation of the triality trace; the global theorem uses a separate residual and relative local triality input.

As a concrete consequence, we obtain a complete description of the automorphisms of the hyperbolic special unitary group $\mathrm{SU}_4(R, \theta; Q_4)$ over every commutative ring with $2 \in R^\times$.

The results are joint work with Deep H. Makadiya.

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